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# Review Advances in studies on two-phase turbulence in dispersed multiphase flows

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#### 1. Introduction

Turbulent dispersed multiphase flows, including gas-particle, gas-droplet, liquid-particle and bubble-liquid flows, are widely encountered in power, chemical, metallurgical, aeronautical, astronautical, nuclear and hydraulic engineering. The turbulence of fluid (gas or liquid) itself is already a complex phenomenon. Turbulent dispersed multiphase flows with co-existing dispersed phase (particles, droplets, bubbles) and continuous phase (gas or liquid) are much more complex. The particles, droplets or bubbles have their own strong fluctuations leading to their dispersion (diffusion), and meanwhile the existence of the dispersed phases will cause the change (modification) of the fluid turbulence. There are strong turbulence interactions between the dispersed and continuous phases. The turbulent fluctuation of the dispersed phase will affect its mixing with the continuous phase, hence has important effect on the pressure drop, heat and mass transfer between two phases, collection efficiency, flame stabilization, combustion efficiency, pollutant formation, etc. There was less understanding to the behavior of so-called particle/bubble/droplet turbulence until the second half of years 80 of the last century. Over a long period the most popular theory was the Hinze–Tchen's ''particle-trackingfluid" theory [\(Hinze, 1975](#page-7-0)), according to which the particle turbulent fluctuation should be always weaker than the fluid turbulent fluctuation, and the larger the particle size, the weaker its turbulent fluctuation. In the framework of two-fluid models, [Elghobashi](#page-7-0) [et al. \(1984\)](#page-7-0) combined the gas  $k$ - $\varepsilon$  turbulence model with an algebraic particle turbulence model (it is called by us a  $k-\varepsilon-A_p$  model). Similar approaches have been taken by [Melville and Bray \(1979\),](#page-7-0) [Chen and Wood \(1985\), Mostafa and Mongia \(1988\),](#page-7-0) etc. All of

#### ABSTRACT

Particle/droplet/bubble fluctuation and dispersion are important to mixing, heat and mass transfer, combustion and pollutant formation in dispersed multiphase flows, but are insufficiently studied before the 90 years of the last century. In this paper, the present author reports his systematic studies within nearly 20 years on two-phase turbulence in dispersed multiphase flows, including particle fluctuation in dilute gas-particle and bubble-liquid flows, particle-wall collision effect, coexistence of particle turbulence and inter-particle collisions, fluid turbulence modulation due to the particle wake effect and validation of the two-fluid RANS modeling using large-eddy simulation.

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**Multiphase Flow** 

these approaches for the particle turbulence are based on the idea of Hinze–Tchen's particle-tracking-fluid theory of particle fluctuation. However, it was found by the present author that in some cases or in some regions of the flow field, in contrast to the Hinze–Tchen's theory, the particle fluctuation is stronger than the fluid turbulent fluctuation, and the larger the particle size, the stronger its turbulent fluctuation. Instead of Hinze–Tchen's theory, a transport equation theory of particle turbulent kinetic energy was proposed [\(Zhou and Huang, 1990](#page-8-0)), according to which particle turbulent fluctuation depends on its own convection, diffusion, production due to mean motion and dissipation/production due to the effect of fluid turbulence, and not only the effect of fluid turbulence, as that predicted by the Hinze–Tchen's theory. Subsequently, [Tu \(1995\)](#page-7-0) also proposed a transport equation of particle turbulent kinetic energy, similar to that proposed by Zhou and Huang with only minor difference in the closure models of some phase interaction terms.

Later, it was found that the anisotropy of particle turbulence is even greater than that of fluid turbulence. A unified second-order moment (USM) theory, i.e., a theory of two-phase Reynolds stress transport equations, was proposed ([Zhou et al., 1994; Zhou and](#page-8-0) [Chen, 2001\)](#page-8-0). On the other hand, a group of investigators, for example, [Zaichik \(2001\), Reeks \(1992\), Simonin \(1996\),](#page-7-0) derived and closed the particle Reynolds stress equations based on the probability density function (PDF) approach. Due to the limitation of the length of this paper, in the following text only the two-phase turbulence models developed by Zhou et al. will be reviewed.

For the effect of wall on particle flow behavior a particle-wall collision theory accounting for the friction, restitution and wall roughness was proposed [\(Zhang and Zhou, 2005](#page-8-0)). For dense gas-particle flows, both large-scale fluctuation due to particle turbulence and small-scale fluctuation due to inter-particle collision are taken into account using a so-called USM- $\Theta$  theory ([Yu and Zhou et al., 2005\)](#page-7-0).



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<span id="page-1-0"></span>For the effect of particles on gas turbulence, including the particle wake effect, it was studied using both LES and RANS modeling [\(Zeng](#page-8-0) [and Zhou et al., 2007\)](#page-8-0). Also, to understand the instantaneous turbulence structures, large-eddy simulation of liquid-bubble flows was carried out ([Yang et al., 2002\)](#page-7-0). Recently, the gas-particle flows are studied using LES, in order to use LES statistical results for validating the USM closure models.

In the following text, we will give a brief description and experimental verification of the proposed models: (a) the two-phase Reynolds stress equation (unified second-order moment, USM) and two-phase turbulent kinetic energy equation  $(k-\varepsilon-k_p)$  models; (b) the two-phase particle-wall collision model; (c) the USM model for dense gas-particle flows (USM- $\Theta$  model); (d) the gas turbulence modification model accounting the particle wake effect; (e) a twofluid large-eddy simulation (LES) of gas-particle flows.

#### 2. Particle fluctuation and dispersion in dilute gas-particle flows

For predicting particle fluctuation, Tchen first considered the single-particle motion in a fluid eddy, and afterwards Hinze used the Taylor's statistical theory of turbulence to obtain the Hinze– Tchen's model [\(Hinze, 1975\)](#page-7-0) for the ratio of particle viscosity over gas viscosity or particle diffusivity over gas diffusivity as

$$
\begin{aligned} v_p / v_T &= D_p / D_T = (k_p / k)^2 = (1 + \tau_{r1} / \tau_T)^{-1}; \\ \tau_{r1} &= \rho_s d_p^2 / (18 \mu), \quad \tau_T = k / \varepsilon \end{aligned} \tag{1}
$$

This model can simply be denoted as an " $A<sub>p</sub>$  model". It is used together with the gas turbulence  $k$ - $\varepsilon$  model, constituting a  $k$ - $\varepsilon$ - $A$ <sub>p</sub> model, and even nowadays is widely adopted as particle dispersion models in two-fluid models in some commercial software. According to Eq. (1), the particle fluctuation should be always smaller than the gas fluctuation and the larger the particle size, the smaller the particle fluctuation. However, in contrast to what predicted by the  $A<sub>p</sub>$  model, the LDV and PDPA measurements show that the particle turbulence intensity is larger than the gas one in the whole flow field of confined jets and in the reverse flow zones of recirculating and swirling flows, and the particle turbulence intensity increases with the increase of the particle size in a certain size range. Based on the concept of transport of particle turbulence, we started from the fluid N–S equation and instantaneous particle motion equation, using the Reynolds expansion and time averaging, derived and closed an energy equation model of particle turbulence ( $k<sub>p</sub>$  model) ([Zhou and Huang, 1990](#page-8-0)). In 1990–1994 we proposed a two-phase Reynolds stress transport equation model, i.e., a unified second-order moment (USM) twophase turbulence model ([Zhou et al., 1994\)](#page-8-0). Based on two-phase instantaneous momentum equations, using Reynolds expansion and time averaging, the fluid and particle Reynolds stress equations are derived and closed. In this case the governing equations for isothermal turbulent gas-particle flows, accounting for only the gravitational and drag forces, including fluid and particle continuity, momentum and Reynolds stress equations, can be given as:

$$
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j} (\rho V_j) = 0 \tag{2}
$$

$$
\frac{\partial \rho_p}{\partial t} + \frac{\partial}{\partial x_j} (\rho_p V_{pj}) = 0
$$
\n(3)

$$
\frac{\partial}{\partial t}(\rho V_i) + \frac{\partial}{\partial x_j}(\rho V_j V_i) = -\frac{\partial p}{\partial x_i} + \frac{\partial \tau_{ji}}{\partial x_j} + \Delta \rho g_i + \frac{\rho_p}{\tau_{rp}}(V_{pi} - V_i)
$$
(4)

$$
\frac{\partial}{\partial t}(\rho_p V_{pi}) + \frac{\partial}{\partial x_j}(\rho_p V_{pj} V_{pi}) = \rho_p g_i + \frac{\rho_p}{\tau_{rp}}(V_i - V_{pi})
$$
\n(5)

$$
\frac{\partial}{\partial t}(\rho \overline{v_i v_j}) + \frac{\partial}{\partial x_k}(\rho V_k \overline{v_i v_j}) = D_{ij} + P_{ij} + G_{p_{ij}} + \Pi_{ij} - \varepsilon_{ij}
$$
(6)

$$
\frac{\partial}{\partial t}(N_p \overline{\nu_{pi} \nu_{pj}}) + \frac{\partial}{\partial x_k}(N_p V_{pk} \overline{\nu_{pi} \nu_{pj}}) = D_{p,ij} + P_{p,ij} + \varepsilon_{p,ij}
$$
(7)

where,  $D_{ij}, P_{ij}, \Pi_{ij}, \varepsilon_{ij}$  are terms having the same meanings as those well known in single-phase fluid Reynolds stress equations. The new source term for two-phase flows

$$
G_{p,ij} = \sum_{p} \frac{\rho_p}{\tau_{rp}} (\overline{\nu_{pi} \nu_j} + \overline{\nu_{pj} \nu_i} - 2 \overline{\nu_i \nu_j})
$$

is a phase interaction term expressing the fluid Reynolds stress production/destruction due to particle drag force. The transport equation of dissipation rate of fluid turbulent kinetic energy for two-phase flows is:

$$
\frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_k}(\rho V_k \varepsilon) = \frac{\partial}{\partial x}\left(c_\varepsilon \frac{k}{\varepsilon} \overline{\nu_k \nu_l} \frac{\partial \varepsilon}{\partial x_l}\right) + \frac{\varepsilon}{k} \left[c_{\varepsilon 1}(G + G_p) - c_{\varepsilon 2}\rho \varepsilon\right]
$$
(8)

where the new source term is  $G_p = \sum_p \frac{\rho_p}{\tau_{rp}} (\overline{\nu_{pi}} \overline{\nu_i} - \overline{\nu_i} \overline{\nu_i}).$ 

 $D_{p,ij}, P_{p,ij}, \varepsilon_{p,ij}$  are the diffusion, production terms of particle Reynolds stress and the production term due to fluid turbulence, respectively

For a closed system, beside Eqs. (6)–(8), the transport equations of  $\overline{n_p \nu_{pi}}, \overline{n_p \nu_{pi}}, \overline{n_p \overline{n_p}}, \overline{\nu_{pi} \nu_i}, \overline{\nu_{pi} \nu_i}$  also should be used. For example, the transport equations of two-phase velocity correlation  $\overline{v_{ni}v_i}$ and particle turbulent kinetic energy are derived based on the fluid and particle momentum equations and closed as:

$$
\frac{\partial}{\partial t} (\overline{\nu_{pi}} \overline{\nu_{j}}) + (V_{k} + V_{pk}) \frac{\partial}{\partial x_{k}} (\overline{\nu_{pi}} \overline{\nu_{j}})
$$
\n
$$
= \frac{\partial}{\partial x_{k}} \left[ (v_{e} + v_{p}) \frac{\partial}{\partial x_{k}} (\overline{\nu_{pi}} \overline{\nu_{j}}) \right] + \frac{1}{\rho \tau_{rp}} \left[ \rho_{p} \overline{\nu_{pi}} \overline{\nu_{pi}} + \rho \overline{\nu_{i}} \overline{\nu_{j}} - (\rho + \rho_{p}) \overline{\nu_{pi}} \overline{\nu_{j}} \right] - \left( \overline{\nu_{pk} \nu_{j}} \frac{\partial V_{pi}}{\partial x_{k}} + \overline{\nu_{k} \nu_{pi}} \frac{\partial V_{j}}{\partial x_{k}} \right) - \frac{\varepsilon}{k} \overline{\nu_{pi} \nu_{j}} \delta_{ij}
$$
\n(9)

$$
\frac{\partial}{\partial t}(N_p k_p) + \frac{\partial}{\partial x_k}(N_p V_{pk} k_p) = \frac{\partial}{\partial x_k}\left(N_p c_p^s \frac{k_p}{\varepsilon_p} \frac{\partial k_p}{\partial p_k \nu_{pl}} \frac{\partial k_p}{\partial x_l}\right) + P_p - N_p \varepsilon_p \tag{10}
$$

where the last term on the right-hand side of Eq. (9) is closed by assuming that the dissipation of two-phase velocity correlation is proportional to the dissipation rate of the gas turbulent kinetic energy.

and 
$$
\varepsilon_p = -\frac{1}{\tau_{rp}} \left[ \overline{\nu_{pi} \nu_i} + \overline{\nu_{pi} \nu_{pi}} + \frac{1}{N_p} (V_i - V_{pi}) \overline{n_p \nu_{pi}} \right]
$$
.

Eqs. (6)–(10) constitute the unified second-order moment twophase turbulence model. It is found that the  $k-\varepsilon-k_p$  model is a reduced form of the USM model in case of nearly isotropic turbulent gas-particle flows, which consists of the following expressions and equations

$$
\overline{\nu_i \nu_j} = \frac{2}{3} k \delta_{ij} - \nu_t \left( \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right), \overline{\nu_{pi} \nu_{pj}} = \frac{2}{3} k_p \delta_{ij} - \nu_p \left( \frac{\partial V_{pi}}{\partial x_j} + \frac{\partial V_{pj}}{\partial x_i} \right) \tag{11}
$$

$$
\overline{n_p v_{pj}} = -\frac{v_p}{\sigma_p} \frac{\partial N_p}{\partial x_j}, \quad \overline{n_p v_{pj}} = -\frac{v_p}{\sigma_p} \frac{\partial N_p}{\partial x_j}
$$
\n
$$
\frac{\partial}{\partial \phi} \left( u - \frac{\partial k}{\partial x_j} \right) \tag{12}
$$

$$
\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_j}(\rho V_j k) = \frac{\partial}{\partial x_j} \left(\frac{\mu_e}{\sigma_k} \frac{\partial k}{\partial x_j}\right) + G + G_p - \rho \varepsilon
$$
\n(13)

$$
\frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_j}(\rho V_j \varepsilon) = \frac{\partial}{\partial x_j} \left( \frac{\mu_e}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_j} \right) + \frac{\varepsilon}{k} \left[ c_{\varepsilon 1} (G + G_p) - c_{\varepsilon 2} \rho \varepsilon \right]
$$
(14)

$$
\frac{\partial (N_p k_p)}{\partial t} + \frac{\partial}{\partial x_k} (N_p k_p V_{pk}) = \frac{\partial}{\partial x_k} \left( \frac{N_p v_p}{\sigma_p} \frac{\partial k_p}{\partial x_k} \right) + P_p - N_p \varepsilon_p \tag{15}
$$

$$
\frac{\partial}{\partial t}(k_{pg}) + (V_k + V_{pk}) \frac{\partial}{\partial x_k}(k_{pg}) \n= \frac{\partial}{\partial x_k} \left( \left( c_s \frac{k^2}{\varepsilon} + c_{k_p} \frac{k_p^2}{\varepsilon_p} \right) \frac{\partial k_{pg}}{\partial x_k} \right) + \frac{1}{\rho \tau_{rp}} \left( \rho_p k_p + \rho k - (\rho + \rho_p) k_{pg} \right) \n- \frac{1}{2} \left( \overline{v_i v_{pk}} \frac{\partial v_{pi}}{\partial x_k} + \overline{v_{pi}} \frac{\partial v_i}{\partial x_k} \right) - \frac{1}{\tau_e} k_{pg}
$$
\n(16)

The physical meanings of the USM and  $k-\varepsilon-k_p$  models are (1) the particle turbulent fluctuation is determined not only by the local gas turbulence as that given by the  $A_p$  model, but also by its own

<span id="page-2-0"></span>

Fig. 1. Particle number density (after Laslandes and Sacre).

convection, diffusion and production, so in some cases or some regions the particle turbulence may be stronger than the gas turbulence; [\(2\)](#page-1-0) the particle turbulent fluctuation is anisotropic and its anisotropy may be stronger than that of gas turbulence. Fig. 1 shows the simulation results of particle number density in windsand flows behind an obstacle [\(Laslandes and Sacre, 1998](#page-7-0)) using both  $k-\varepsilon-k_p$  and  $k-\varepsilon-A_p$  models and their comparison with experiments. It is seen that the  $k$ - $\varepsilon$ - $A_p$  model based on the theory of particle tracking local gas turbulence, seriously under-predicts the particle dispersion leading to more ununiform particle concentration distribution, not observed in experiments, whereas the  $k$ - $\varepsilon$  $k_p$  model accounting for the convection of particle turbulence, much better predicts the particle dispersion, giving a more uniform particle concentration distribution, in much better agreement with the measurement results. Fig. 2 gives the predicted vertical normal Reynolds stresses for the liquid and bubbles in bubble-liquid flows in a bubble column using a full second-order moment (FSM) model and an algebraic stress model (ASM) [\(Zhou and Yang et al., 2002\)](#page-8-0) and their comparison with the PIV measurement results, indicating a good agreement. The results show that in the bubble column the dispersed phase turbulence-bubble turbulence is much stronger than the liquid turbulence due to its higher inlet velocity, and the liquid (with lower inlet velocity) turbulence is produced not only by its own velocity gradient but also by the enhancement due to bubble fluctuation. The results also indicate that the anisotropy of bubble turbulence is stronger that of liquid turbulence (not shown here). These results are in contrast to what predicted previously by some investigators who told us that bubbles always attenuate liquid turbulence.

#### 3. Particle-wall collision effect

It is well known that the particle-wall collisions are directly treated in the Lagrangian discrete particle simulation. In the early-developed Eulerian–Eulerian or two-fluid modeling of fluidparticle flows, the particle-wall collision was not taken into account, and zero normal particle velocity and zero normal gradient of other particle variables at the wall are assumed as

$$
V_{pw} = 0 \quad \left(\frac{\partial \phi_p}{\partial y}\right)_w = 0 \tag{17}
$$

This model is equivalent to the full reflection condition without energy loss in the Lagrangian approach, which is obviously not true in practical gas-particle flows where particle-wall collision plays important role. A particle-wall collision model in the framework of two-fluid approach, taking the restitution, friction and wall roughness into account was proposed by the present author ([Zhang](#page-8-0) [and Zhou, 2005](#page-8-0)). For example, the particle number density, longitudinal velocity and longitudinal component of normal Reynolds stresses at the walls are given as

$$
N_{pb} = \frac{1}{2} N_{p1} \left( 1 + \frac{1}{e} \right) \left( 1 - \frac{1}{\sqrt{3}} \frac{V_{p1}}{\sqrt{2k_p/3}} \right)
$$
(18)

$$
V_{pb} = (V_{p1} + V_{p1}f)\left(1 - \frac{1}{3}\alpha'^2\right)
$$
 (19)

$$
\overline{u_p u_{pb}} = \frac{1}{3} \overline{u_{p1} u_{p1}} \{ 3 - \alpha'^2 [2 - f^2 (1 + e)] \}
$$
  
+ 
$$
\frac{1}{3} \overline{v_{p1} v_{p1}} (1 + e) [3f^2 + \alpha'^2 (1 - 2f^2)]
$$
  
+ 
$$
\frac{2}{3} \overline{u_{p1} v_{p1}} f [3 - \alpha'^2 (2e + 3)] + \frac{1}{3} U_{p1} U_{p1} \alpha'^2 f^2 (1 + e)
$$
  
+ 
$$
\frac{1}{3} V_{p1} V_{p1} [3ef^2 + \alpha'^2 (1 + e - 2ef^2)] - \frac{2}{3} U_{p1} V_{p1} \alpha'^2 f (1 + 2e)
$$
(20)

where  $f$ ,  $e$  and  $\alpha$  denote the friction coefficient, restitution coefficient and wall roughness, respectively, the capital alphabets U and V denote time-averaged particle velocities and lower-case alphabets  $u$  and  $v$  denote particle fluctuation velocities, the subscript *b* denotes the values at the wall, and the subscript 1 denotes the values in the near-wall grid nodes. These equations imply that



Fig. 2. Vertical normal reynolds stress ( $\bullet$  Exp.;  $-$  FSM; ... ASM).

<span id="page-3-0"></span>

Fig. 3. Particle tangential time-averaged velocity (m/s).



Fig. 4. Particle RMS tangential fluctuation velocity.

the particle number density,velocity components and Reynolds stresses will change under the effect of particle-wall collision due to friction, restitution and wall roughness, and not obey the law of zero normal velocity and zero-gradient of other variables. The wall roughness can lead to redistribution of particle Reynolds stress components after particle-wall collision. The predicted particle tangential time-averaged velocity (Fig. 3) and RMS tangential fluctuation velocity (Fig. 4) of swirling gas-particle flows measured by [Sommerfeld and Qiu \(1991\)](#page-7-0) show that the prediction results using the boundary condition ''bc 2", based on Eqs. [\(18\)–\(20\),](#page-2-0) give lower near-wall particle tangential time-averaged and RMS fluctuation velocities due to the effect particle-wall collisions, in agreement with those observed in experiments, whereas the prediction results using the boundary condition "bc 1", based on Eq. [\(17\),](#page-2-0) not accounting for the particle-wall collisions, give higher near-wall particle time-averaged and RMS fluctuation velocities, not in agreement with experimental results.

#### 4. Coexistence of particle turbulence and inter-particle collisions

In dense gas-particle flows there are both large-scale particle fluctuations due to particle turbulence and small-scale particle fluctuations due to inter-particle collisions. A USM- $\Theta$  two-phase turbulence model for dense gas-particle flows was proposed by the present author [\(Yu and Zhou et al., 2005\)](#page-7-0). In this model the gas turbulence and particle large-scale fluctuation are predicted using the USM two-phase turbulence model, and the particle small-scale fluctuation due to inter-particle collisions is predicted using the particle pseudo-temperature equation –  $\Theta$  equation, given by Gidaspow's kinetic theory ([Gidaspow, 1994](#page-7-0)). This is not a simple superposition, since there are interaction terms in the particle Reynolds stress equations and the  $\Theta$  equation. Some of the closed USM- $\Theta$  model equations are:

The gas Reynolds stress equation  $\overline{ }$ 

 $\Delta$ 

$$
\frac{\partial \left(\overline{\alpha_g} \rho_{gm} \overline{v_{gi} v_{gi}}\right)}{\partial t} + \frac{\partial \left(\overline{\alpha_g} \rho_{gm} V_{gk} \overline{v_{gi} v_{gi}}\right)}{\partial x_k} \n= D_{g,ij} + P_{g,ij} + \Pi_{g,ij} - \varepsilon_{g,ij} + G_{g,gp,ij} \n\text{where } G_{g,gp,ij} = \beta \left(\overline{v_{pi} v_{gj}} + \overline{v_{gi} v_{pj}} - 2 \overline{v_{gi} v_{gj}}\right)
$$
\n(21)

The particle Reynolds stress equation  $\Delta$ 

$$
\frac{\partial \left(\overline{\alpha_p} \rho_{pm} \overline{\nu_{pi} \nu_{pj}}\right)}{\partial t} + \frac{\partial \left(\overline{\alpha_p} \rho_{pm} V_{pk} \overline{\nu_{pi} \nu_{pj}}\right)}{\partial x_k} \n= D_{p,ij} + P_{p,ij} + H_{p,ij} - \varepsilon_{p,ij} + G_{p,gp,ij}
$$
\n(22)

where  $G_{p,gp,ij} = \beta \left( \overline{\nu_{pi} \nu_{gj}} + \overline{\nu_{pj} \nu_{gi}} - 2 \overline{\nu_{pi} \nu_{pj}} \right)$ 

The equations of dissipation rate of turbulent kinetic energy for gas and particle phases:  $\overline{1}$  $\Delta$  $\overline{1}$  $\overline{1}$ 

$$
\frac{\partial \left(\overline{\alpha_g} \rho_{gm} \varepsilon_g\right)}{\partial t} + \frac{\partial \left(\overline{\alpha_g} \rho_{gm} V_{gk} \varepsilon_g\right)}{\partial x_k} \n= \frac{\partial}{\partial x_k} \left(C_g \overline{\alpha_g} \rho_{gm} \frac{k_g}{\varepsilon_g} \overline{\nu_{gk} \nu_{gl} \frac{\partial \varepsilon_g}{\partial x_l}}\right) + \frac{\varepsilon_g}{k_g} \left[c_{\varepsilon 1} \left(P_g + G_{g,gp}\right) - c_{\varepsilon 2} \overline{\alpha_g} \rho_{gm} \varepsilon_g\right]
$$
\n(23)

where 
$$
G_{g,gp} = 2\beta(k_{gp} - k_g)
$$
,  $c_{e3} = 1.8$   
\n
$$
\frac{\partial (\overline{\alpha_p} \rho_{pm} \epsilon_p)}{\partial t} + \frac{\partial (\overline{\alpha_p} \rho_{pm} V_{pk} \epsilon_p)}{\partial x_k}
$$
\n
$$
= \frac{\partial}{\partial x_k} (\overline{\alpha_p} \rho_{pm} C_p^d \frac{k_p}{\epsilon_p} \frac{\overline{\nu_{pk}} \nu_{pl} \partial \epsilon_p}{\partial x_l}) + \frac{\epsilon_p}{k_p} [C_{ep,1} (P_p + G_{p,gp}) - C_{ep,2} \overline{\alpha_p} \rho_{pm} \epsilon_p]
$$
\n(24)

where  $G_{p,gp} = 2\beta(k_{pg} - k_p)$ 





The two-phase velocity correlation equation:

$$
\frac{\partial \overline{v_{pi}v_{gj}}}{\partial t} + (V_{gk} + V_{pk}) \frac{\partial \overline{v_{pi}v_{gj}}}{\partial x_k} = D_{g,p,ij} + P_{g,p,ij} + \Pi_{g,p,ij} - \varepsilon_{g,p,ij} + T_{g,p,ij}
$$
\n(25)

The particle pseudo-temperature transport equation:

$$
\frac{3}{2} \left[ \frac{\partial \left( \overline{\alpha_p} \rho_{pm} \Theta \right)}{\partial t} + \frac{\partial \left( \overline{\alpha_p} \rho_{pm} V_{pk} \Theta \right)}{\partial x_k} \right]
$$
\n
$$
= -\frac{\partial}{\partial x_k} \left( \frac{3}{2} \frac{\partial \rho_{pm} \rho_{pm} \Theta}{\partial \rho_{pm} \rho_{pk} \Theta} + \Gamma_{\Theta} \frac{\partial \Theta}{\partial x_k} \right) + \mu_p \left( \frac{\partial V_{pk}}{\partial x_i} + \frac{\partial V_{pi}}{\partial x_k} \right) \frac{\partial V_{pi}}{\partial x_k}
$$
\n
$$
+ \mu_p \varepsilon_p - P_p \frac{\partial V_{pl}}{\partial x_l} + \left( \xi_p - \frac{2}{3} \mu_p \right) \left( \frac{\partial V_{pl}}{\partial x_l} \right)^2 - \overline{\gamma}
$$
\n(26)

The interaction between the large-scale and small-scale particle fluctuations is the third term on the right-hand side of Eq. (26), expressing the effect of the dissipation rate of particle turbulent kinetic energy on the particle pseudo-temperature. Figs. 5 and 6 give the simulation results of particle volume fraction (Fig. 5) and particle velocity (Fig. 6), respectively, for dense gas-particle flows in a downer, measured by [Wang et al. \(1992\)](#page-7-0). It is seen that the USM- $\Theta$  model, accounting for both particle turbulence and interparticle collision, gives the particle volume fraction and velocity distribution in best agreement with the measurement results. The  $DSM-\Theta$  model, neglecting particle turbulence, gives a high peak of particle volume fraction near the wall and non-uniform particle velocity distribution, not observed in experiments. The USM model, neglecting inter-particle collision, gives too uniform particle volume fraction and velocity distributions, not in agreement with experiments. The  $k-\varepsilon-k_p-\Theta$  model, neglecting the anisotropy of particle turbulence, also over-predicts the non-uniformness of particle velocity distribution. Figs. 7 and 8 show the simulation results of particle horizontal and vertical RMS fluctuation velocities for horizontal gas-particle pipe flows measured by [Kussin and Sommerfeld](#page-7-0) [\(2002\)](#page-7-0). It is seen that the USM- $\Theta$  model can more properly predict the anisotropy of particle RMS fluctuation velocities-the axial component is greater than the vertical component, whereas the USM model over-predict this anisotropy and the  $k-\epsilon-k_n-\Theta$  model entirely cannot predict this anisotropy.

#### 5. Turbulence modulation in gas-particle flows with the particle wake effect

The problem of gas turbulence in gas-particle flows or, so-called turbulence modulation from the single-phase turbulence, attracts more and more attention in recent years. For dilute gas-particle



Fig. 6. Particle velocity.



Fig. 7. Particle horizontal RMS fluctuation velocity.



Fig. 8. Particle vertical RMS fluctuation velocity.

flows, various empirical and semi-empirical models have been proposed. Up to now, in most of DNS, LES and RANS modeling, the particles are treated as point sources. In the two-fluid approach of RANS modeling, the particle-source term in the gas Reynolds stress equation or the turbulent kinetic energy equation is the difference between the gas-particle velocity correlation and the gas Reynolds stress. Owing to the fact that the former is always smaller than the latter, the obtained source term is always negative, leading to the



Fig. 9. Air turbulence intensity ((a) with 0.5 mm particles; (b) with 1 mm particles; (c) with 0.2 mm particles).

dissipation of gas turbulence. In fact, particle has finite volume. When gas passes over the particle, both the wake behind the particle and the vortex shedding should contribute to the velocity disturbance and are considered as the sources of turbulence production. Since the large-size eddies are mainly responsible for the mechanism of particle enhancing gas turbulence, in our studies [\(Zeng and](#page-8-0) [Zhou et al., 2007\)](#page-8-0) we at first did the simulation of gas turbulent flows passing a single particle using both RANS modeling with a Reynolds stress equation turbulence model and LES with a Smagorinsky subgrid scale stress model. The turbulence enhancement by the particle is studied under various particle sizes and relative gas velocities. Based on these simulation results, a turbulence enhancement model for the particle wake effect is proposed. Then, the proposed model is taken as a sub-model, incorporated into the two-phase flow model, i.e., the second-order moment two-phase turbulence model and is used to simulate dilute gas-particle flows. The simulation results are compared with experimental results and the simulation results obtained by the two-phase flow model not accounting for the particle wake effect.

A turbulence enhancement model in the gas Reynolds stress equation based on the single-particle simulation is obtained as

$$
G_{pw} = c \frac{\rho_p \alpha_p V_{rel}^2}{\tau_{rp}} \tag{27}
$$

where

$$
\tau_{rp} = \frac{\rho_p d_p^2}{18\mu_g (1 + Re_p^{2/3}/6)}, \quad Re_p = \frac{\alpha_g \rho_g d_p \left| \vec{V_g} - \vec{V_p} \right|}{\mu_g}
$$

Eq. [\(23\)](#page-3-0) indicates that the turbulence enhancement due to the particle wake effect is proportional to the particle size and the squire of relative velocity. The gas Reynolds stress equation with the particlesource term accounting for the particle wake effect is:

$$
\frac{\partial \left(\overline{\alpha_g} \rho_{gm} \overline{\nu_{gi} \nu_{gi}}\right)}{\partial t} + \frac{\partial \left(\overline{\alpha_g} \rho_{gm} V_{gk} \overline{\nu_{gi} \nu_{gi}}\right)}{\partial x_k} \n= D_{g,ij} + P_{g,ij} + \Pi_{g,ij} - \varepsilon_{g,ij} + G_{g,gp,ij} + G_{pw} \delta_{ij}
$$
\n(28)

Fig. 9 gives the RMS gas fluctuation velocities with different sizes of particles in vertical gas-particle pipe flows, measured by [Tsuji et al.](#page-7-0) [\(1984\)](#page-7-0). It is found that the results obtained using the model accounting for the particle wake effect are in much better agreement with the experimental results than those obtained using the model not accounting for the particle wake effect in predicting the following phenomena: 1 mm particles only enhance gas turbulence intensity, 0.5 mm particles enhance or attenuate gas turbulence at different locations, and 0.2 mm particles only attenuate gas turbulence.

#### 6. A two-fluid LES of gas-particle flows and validation of the USM two-phase turbulence model

Large-eddy simulation (LES) can give us the instantaneous turbulence structures and its statistical results can be used to validate the RANS turbulence models. LES is used by us to validate the USM two-phase turbulence model. The filtered governing equations for a two-fluid LES are given as

$$
\frac{\partial}{\partial t}(\alpha_k \rho_k) + \frac{\partial}{\partial x_j}(\alpha_k \rho_k V_{kj}) = 0 \quad (k = g, p)
$$
\n
$$
\frac{\partial}{\partial t}(\rho_k V_{kj}) = 0 \quad (k = g, p) \tag{29}
$$

$$
\frac{\partial}{\partial t} \left( \alpha_g \rho_g V_{gi} \right) + \frac{\partial}{\partial x_j} (\alpha_g \rho_g V_{gi} V_{gi}) \n= -\frac{\partial p_g}{\partial x_j} + \frac{\partial \tau_{g,sgsj}}{\partial x_j} + \frac{\partial \tau_g}{\partial x_j} + \frac{\alpha_g \rho_g}{\tau_r} (v_{pi} - v_{gi})
$$
\n(30)

$$
\frac{\partial}{\partial t} (\alpha_p \rho_p V_{pi}) + \frac{\partial}{\partial x_j} (\alpha_p \rho_p V_{pj} V_{pi})
$$
\n
$$
= -\frac{\partial \tau_s}{\partial x_j} + \frac{\alpha_s \rho_s}{\tau_r} (V_{gi} - V_{pi}) + \frac{\partial \tau_{p,sgs}}{\partial x_j}
$$
\n(31)

For particle-collision stress, neglecting the sub-grid scale particle stress and using the particle pseudo-temperature proposed by Gidaspow's kinetic theory [\(Gidaspow, 1994\)](#page-7-0)

$$
\tau_p = \left\{-\alpha_p \rho_p \Theta \big[1+2(1+e)g_0 \alpha_p\big] + \alpha_p p_p \nabla \cdot V_p\right\} \delta_{ij} - 2 \alpha_p \mu_p S_p \quad (32)
$$

$$
\frac{3}{2} \left[ \frac{\partial}{\partial t} \left( \alpha_p \rho_p \Theta \right) + \nabla \cdot \left( \alpha_p \rho_s \Theta V_s \right) \right] = \nabla \cdot \left( \kappa_p \nabla \Theta \right) - \gamma_p - 3\beta \Theta \quad (33)
$$



**Fig. 10.** Particle RMS fluctuation velocities (m  $s^{-1}$ ;  $\blacksquare$  Exp.,  $-$  LES, ... USM).

The Smagorinsky model ([Smagorinsky, 1963\)](#page-7-0) is used for the gas sub-grid scale stress

$$
\tau_{g,sgs,ij} = -2\nu_{T}\overline{S}_{ij} + \frac{1}{3}\delta_{ij}\tau_{kk}; \quad \nu_{T} = C_{s}^{2} \Delta^{2}|\overline{S}|; \quad \overline{S}_{ij} = (\partial V_{i}/\partial x_{j} + \partial V_{j}/\partial x_{i})/2
$$
  

$$
|\overline{S}| = (2\overline{S}_{ij}\overline{S}_{ij})^{1/2}
$$
 (34)

Fig. 10 gives the LES and USM simulated particle RMS fluctuation velocities and their comparison with experimental results for backward-facing step gas-particle flows ([Hishida and Maeda, 1991\)](#page-7-0). Both of these modeling results are in agreement with the experimental results. It implies that the USM two-phase turbulence model is validated by LES.

[Fig. 11](#page-7-0) shows the predicted particle axial RMS fluctuation velocity using LES and USM for axi-symmetric sudden-expansion gasparticle flows measured by [Xu and Zhou \(1999\).](#page-7-0) Both modeling results are in good agreement with experimental results. LES results are somewhat better than the USM results.

[Fig. 12](#page-7-0) shows the axial component of gas-particle velocity correlation. The two models give the same trend in agreement with experiments. The distribution of gas-particle velocity correlation is similar to that of particle axial RMS fluctuation velocity, but the former is smaller than the latter. It is seen that LES results are closer to the experimental results than the USM results. It implies that the USM model remains to be improved. There is still certain discrepancy between the LES results and experiments due to the 2- D LES and the shortcomings of the Smagorinsky SGS stress model, which also should be improved.

#### 7. Conclusions

(1) The USM and  $k-\varepsilon-k_p$  two-phase turbulence models can well predict stronger particle fluctuation than the gas fluctuation and stronger anisotropy of particle turbulence than that of gas turbulence for some cases and in some regions of the flow field. These models are more reasonable than the traditional Hinze–Tchen's theory. However, in some cases, for example, swirling gas-particle flows, the particle RMS fluctuation velocities are still under-predicted.

<span id="page-7-0"></span>![](_page_7_Figure_1.jpeg)

Fig. 11. Particle axial RMS fluctuation velocity.

![](_page_7_Figure_3.jpeg)

Fig. 12. Gas-particle velocity correlation  $m s^{-1}$ .

- (2) The particle-wall collision, including friction, restitution and wall roughness, has important effect on the near-wall particle velocity and turbulence. It gives reduced particle velocity and particle turbulence owing to energy losses during collision. The wall roughness will increase the longitudinal component of particle RMS fluctuation velocity and reduce the normal component, that is, leads to redistribution of normal components of particle normal stresses near the wall.
- (3) In dense gas-particle flows, there is interaction between the large-scale particle fluctuation due to particle turbulence and small-scale particle fluctuation due to inter-particle collision. The former leads to enhancing particle dispersion, whereas the latter will reduce particle large-scale fluctuation. The USM- $\Theta$  model can better predict these phenomena than other models, neglecting either particle turbulence or interparticle collision or the anisotropy of particle turbulence.
- (4) The particle wake effect plays important role in the gas turbulence modulation. The proposed model can well predict different behavior of different size range particles in turbulence modulation. However, the gas turbulence modulation in dense gas-particle flows remains to be further studied.
- (5) The USM two-phase turbulence model is only preliminarily validated by a two-fluid LES. However, a more advanced two-phase sub-grid scale stress model remains to be developed.

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#### References

Chen, C.P., Wood, P.E., 1985. A Turbulence closure model for dilute gas-particle flows. Can. J. Chem. Eng. 63, 349–360.

- Elghobashi, S.E., Abou-Arab, T.W., Risk, M., Mostafa, A., 1984. Prediction of the particle-laden jet with a two-equation turbulence model. Int. J. Multiphase Flow 10, 697–710.
- Gidaspow, D., 1994. Multiphase Flow and Fluidization: Continuum and Kinetic Theory Descriptions. Academic Press.
- Hinze, J.O., 1975. Turbulence. McGraw Hill, New York.
- Hishida, K., Maeda, M., 1991. Turbulence characteristics of particle-laden flow behind a rearward facing step. ASME-FED 121, 207–212.
- Kussin, J., Sommerfeld, M., 2002. Experimental studies on particle behavior and turbulence modification in horizontal channel flow with different wall roughness. Exp. Fluids 33 (1), 143–159.
- Laslandes, S., Sacre, C., 1998. Transport of particles by a turbulent flow around an obstacle-a numerical and a wind tunnel approach. J. Wind Eng. Ind. Aerodyn. 74–76, 577–587.
- Melville, W.K., Bray, K.N.C., 1979. A model of the two-phase turbulent jet. Int. J. Heat Mass Transfer 22, 647–656.
- Mostafa, A.A., Mongia, H.C., 1988. On the interaction of particles and turbulent flow. Int. J. Heat Mass Transfer 31, 2063–2075.
- Reeks, M.W., 1992. On the continuum equations for a dispersed particles in nonuniform flows. Phys. Fluids A4, 1290–1303.
- Simonin, O., 1996. Continuum modeling of dispersed two-phase flows. In: Combustion and Turbulence in Two-Phase Flows, Lecture Series 1996-02, von Karman Institute for Fluid Dynamics.
- Smagorinsky, J., 1963. General circulation experiments with the primitive equation I. The basic experiment. Monthly Weather Rev. 91 (3), 99–108.
- Sommerfeld, M., Qiu, H.H., 1991. Detailed measurements in a swirling particulate two-phase flow by a phase-Doppler anemometer. Int. J. Heat Fluid Flow 12, 15–32.
- Tsuji, Y., Morikawa, Y., Shinomi, H., 1984. LDV measurements of air-solid two-phase flow in a vertical pipe. J. Fluid Mech. 139, 417–434.
- Tu, J.Y., Fletcher, C.A.J., 1995. Numerical computation of turbulent gas-particle flow in a 90-degree bend. AIChE J. 41, 2187–2197.
- Wang, Y., Bai, D.R., Jin, Y., 1992. Hydrodynamics of cocurrent downflow circulating fluidized bed (CDCFB). Powder Technol. 70, 271–275.
- Xu, Y., Zhou, L.X., 1999. Experimental studies on two-phase fluctuation velocity correlation in sudden expansion flows, ASME FED, Paper SM1999-7909, San Francisco.
- Yang, M., Zhou, L.X., Fan, L.S., 2002. Large-eddy simulation of bubble-liquid confined jets. Chin. J. Chem. Eng. 10, 381–384.<br>Yu, Y., Zhou, L.X., Wang, B.G., Cai, F.P., 2005. A USM-0 two-phase turbulence model
- for simulating dense gas-particle flows. Acta Mech. Sin. 21, 228–234.
- Zaichik, L.I., Aplichenkov, V.M., 2001. A statistical model for transport and deposition of high-inertia colliding particles in turbulent flow. Int. J. Heat Fluid Flow 22, 365–371.
- <span id="page-8-0"></span>Zeng, Z.X., Zhou, L.X., Qi, H.Y., 2007. Large eddy simulation of particle wake effect and RANS simulation of turbulence modulation in gas-particle flows. Chin. J. Chem. Eng. 15, 12–16.
- Zhang, X., Zhou, L.X., 2005. A second-order moment particle-wall collision model accounting for the wall roughness. Powder Technol. 159, 111–120. Zhou, L.X., Chen, T., 2001. Simulation of strongly swirling gas-particle
- flows using USM and  $k-\varepsilon-k_p$  two-phase turbulence models. Powder Technol. 114, 1–11.
- Zhou, L.X., Huang, X.Q., 1990. Prediction of confined gas-particle jets by an energy
- equation model of particle turbulence. Sci. Chin. Engl. Ed. 33, 53–59. Zhou, L.X., Liao, C.M., Chen, T., 1994. A unified second-order moment two-phase turbulence model for simulating gas-particle flows, numerical methods in multiphase flows. ASME-FED 185, 307–313.
- Zhou, L.X., Yang, M., Lian, C.Y., Fan, L.S., Lee, D.J., 2002. On the second-order moment turbulence model for simulating a bubble column. Chem. Eng. Sci. 57, 3269– 3281.